**UNIT II – INFINITE IMPULSE RESPONSE FILTERS**

Characteristics of practical frequency selective filters. characteristics of commonly used analog filters - Butterworth filters, Chebyshev filters. Design of IIR filters from analog filters (LPF, HPF, BPF, BRF) - Approximation of derivatives, Impulse invariance method, Bilinear transformation. Frequency transformation in the analog domain. Structure of IIR filter - direct form I, direct form II, Cascade, parallel realizations.

**STRUCTURES FOR IIR SYSTEMS**:

IIR Systems are represented in four different ways

1. Direct Form Structures Form I and Form II

2. Cascade Form Structure

3. Parallel Form Structure

4. Lattice and Lattice-Ladder structure.

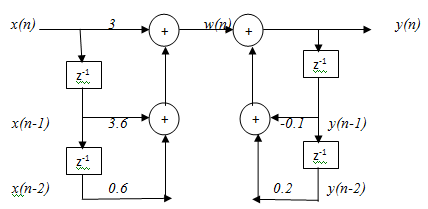
**DIRECT FORM-I :**

***Challenge: Obtain the direct form-I, direct form-II,Cascade and parallel form realization of the system y(n)=-0.1y(n-1)+0.2y(n-2)+3x(n)+3.6x(n-1)+0.6x(n-2) [April/May-2015]***

**Solution:**

**Direct Form I:**





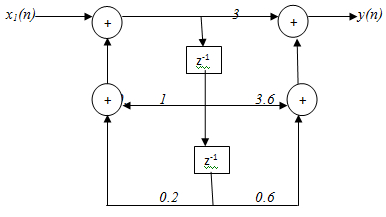
**Direct form II:**

From the given difference equation we have





The above system function can be realized in direct form II



**Cascade Form:**



3

z-1

*x(n) y(n)*

z-1

*-0.5 0.6 0.4*

**Parallel form:**



z-1

z-1

*x(n) -3 7*

*0.4*

*-1**y(n)*

-0.5

**-----------------------------------------------------------------------------------------------------------**

**Direct form I:**

**H.W: Obtain the direct form-I realization for the system described by the following difference equations.**

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**Obtain the direct form-I realization for the system described by difference equation **

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**Direct form II**

**H.W: Determine the direct form II realization for the following system [May/June-14]**

**Determine the direct form II realization for the following system **

**CASCADE FORM:**

**-----------------------------------------------------------------------------------------------------------------**

**H.W: For the system function obtain cascade structure.**

**Realize the system with difference equation** **in cascade form.**

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**Parallel form:**

**H.W: Realize the system given by difference equation**

**in parallel form.**

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**Analog filter design:**

There are two types of analog filter design are,

* + - Butterworth Filter
    - Chebyshev Filter.

***Analog Low pass Butterworth Filter:***

|  |  |
| --- | --- |
| N | Denominator of H(s) |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |

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**Design an analog Butterworth filter that has a -2dB pass band attenuation at a frequency of 20 rad/sec and atleast -10 dB stop band attenuation at 30 rad/sec.**

**Solution:**

Given data:

Pass band attenuation αP= 2 dB;

Stop band attenuation αS= 10 dB;

Pass band frequency ΩP= 20 rad/sec.

Stop band frequency ΩS=30 rad/sec.

The order of the filter





Rounding off ‘N’ to the next higher integer, we get

*N=4*

The normalized transfer function for *N=4.*



To find cut off frequency





The transfer function for Ωc=21.3868,





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**H.W: Challenge 1: For the given specification design an analog Butterworth filter**



**Ans: **

**Challenge 2: Determine the order and the poles of low pass Butterworth filter that has 3 dB attenuation at 500 Hz and an attenuation of 40dB at 1000Hz.**

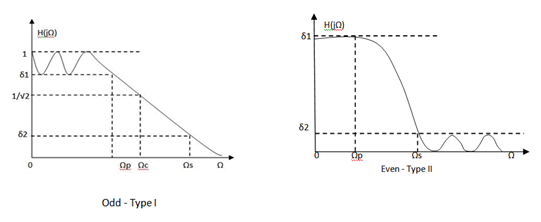
**Ans:** 

**Given the specification determine the order of the filter. Ans: *N=4***

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**Analog Low pass Chebyshev Filter:**

There are two types of Chebyshev filters.

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**Given specifications αp= 3dB,αs=16 dB, fP=1KHz and fS=2KHz. Determine the order of the filter using Chebyshev approximation. Find H(s).**

**Solution:**

Given:

**Step 1:**

Pass band attenuation αp= 3dB,

Stop band attenuation αs=16 dB,

Pass band frequency fP=1 KHz=2π\*1000=2000π rad/sec

Stop band frequency fS=2 KHz=2π\*2\*1000=4000π rad/sec

**Step 2: Order of the filter**





Rounding the next higher integer value *N=2.*

**Step 3: The value of minor axis and major axis can be found as below**

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**Step 4: The poles are given by**







**Step 5: The denominator of *H(s):***

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**Step 6: The numerator of *H(s):***

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**The transfer function **

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**HW: Challenge 1: Obtain an analog Chebyshev filter transfer function that satisfies the constraints**

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**Ans:**

**2. Design a Chebyshev filter with a maximum pass band attenuation of 2.5dB at ΩP=20rad/sec and stop band attenuation of 30 dB at ΩS=50rad/sec.**

**Ans: *N=3. ***

***---------------------------------------------------------------------------------------------------------------------------------***

***3. For the given specifications find the order of the Chebyshev-I filter***

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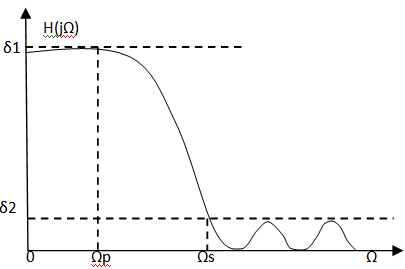
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***4. For the given specifications find the order of the Chebyshev-I filter***

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**Discrete time IIR filter from analog filter:**

**Magnitude Response of LPF:**



**Design of IIR filters from analog filters:**

The different design techniques available for Iir filter are

1. Approximation of derivates
2. Impulse invariant method
3. Bilinear transformation
4. Matched z-transform techniques.

**Approximation of derivates:**

For analog to digital domain, we get

-------------------------- (3)

--------------------- (4)

**Mapping of the s-plane to the z-plane using approximation of derivatives.**

*------------------------------------------------------------------------------------------------------------------------------*

**Convert the analog BPF with system IIR filter into a digital IIR filter by use of the backward difference for the derivative. [Nov/Dec-2015]**

**Solution:**

Given:





**Design of IIR filter using Impulse Invariance Method:**

**Steps to design a digital filter using Impulse Invariance Method (IIM):**

**Step 1:** For the given specifications, find Ha(s) the Transfer function of an analog filter.

**Step 2:** Select the sampling rate of the digital filter, T seconds per sample.

**Step 3:** Express the analog filter transfer function as the sum of single-pole filter.



**Step 4:** Compute the z-transform of the digital filter by using formula



For high sampling rate,



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**For the analog transfer functionDetermine H (z) using impulse invariant transformation if (a) T=1 second and (b) T=0.1 second. [Nov/Dec-15]**

**Solution:**

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**Design a third order Butterworth digital filter using impulse invariant technique. Assume sampling period *T=1 sec.***

**Solution:**

Given: For N=3, the transfer function of a normalized Butterworth filter is given by

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**Ans:** 

**----------------------------------------------------------------------------------------------------------------------------**

**Apply impulse invariant method and find *H(z) for ***

**Solution:**

Given: The transfer function **

Sampling the function produces



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**Convert analog filter into digital IIR filter using impulse invariant method. [Nov/Dec-2015]**

**Solution:**

Given: Analog filter ****







**Convert analog filter into digital IIR filter whose system function is given above. The digital filter should have (). Use impulse invariant mapping T=1sec.**

**Solution:**

Given: Analogfilter ****





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**H.W: Challenge 1: An analog filter has a transfer function. Design a digital filter equivalent to this using impulse invariant method for T=0.2 sec**. **[Nov/Dec-15]**

Ans : 

**2. An analog filter has a transfer function. Design a digital equivalent to this using impulse invariant method for T=1 sec.**

**3. An analog filter has a transfer function. Design a digital filter equivalent to this using impulse invariant method T=1 sec.**

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**Design of IIR filters using Bilinear Transformation:**

**Steps to design digital filter using bilinear transform technique:**

1. From the given specifications, find prewarping analog frequencies using formula 
2. Using the analog frequencies find *H(s)* of the analog filter.
3. Select the sampling rate of the digital filter, call it T seconds per sample.
4. Substitute  into the transfer function found in step2.

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**Apply bilinear transformation of with *T=1 sec* and find *H(z).[Nov/Dec-13]***

**Solution:**

**Given: The system function **

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**Given *T=1 sec.***

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**Using the bilinear transformation, design a high pass filter, monotonic in pass band with cut off frequency of 1000Hz and down 10dB at 350 Hz. The sampling frequency is 5000Hz. [May/June-16]**

**Solution:**

Given: Pass band attenuation; Stop band attenuation 

Pass band frequency 

Stop band frequency 



Prewarping the digital frequencies, we have





The order of the filter

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*The first order Butterworth filter for ΩC=1 rad/sec is H(s) = 1/S+1*

*The high pass filter for ΩC=ΩP=7265 rad/sec can be obtained by using the transformation.*

**

*The transfer function of high pass filter*

**

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***H.W: 1. Determine H(z) that results when the bilinear transformation is applied to*** *Ha(s)=* **[Nov/Dec-15]** Ans:

2. An analog filter has a transfer function ,design a digital filter using bilinear transformation method.

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**Additional Examples:**

**Design a digital Butterworth filter satisfying the constraints**

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**With *T=1 sec using bilinear transformation. [April/May-2015][May/June-14]***

***Solution:***

*Given data:*

*Pass band attenuation; Pass band frequency;*

*Stop band attenuation; Stops band frequency;*

**Step 1: Specifying the pass band and stop band attenuation in dB.**



**Step2. Choose *T* and determine the analog frequencies (i.e) Prewarp band edge frequency  
**

**Step3. To find order of the filter**





*Rounding the next higher value N=2*

**Step 4: The normalized transfer function**



**Step 5: Cut off frequency**





**Step 6: To find Transfer function of *H(s):***





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**Step 7. Apply Bilinear Transformation with to obtain the digital filter**

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**Design a digital Butterworth filter satisfying the constraints**

****

**With *T=1 sec using Impulse invariant method. [Nov/Dec-13]***

***Solution:***

*Given data:*

*Pass band attenuation; Pass band frequency;*

*Stop band attenuation; Stops band frequency;*

**Step 1: Specifying the pass band and stop band attenuation in dB.**



**Step2. Choose *T* and determine the analog frequencies (i.e) Prewarp band edge frequency  
**

**Step3. To find order of the filter**





*Rounding the next higher value N=4*

**Step 4: The normalized transfer function**



**Step 5: Cut off frequency**





**Step 6: To find Transfer function of *H(s):***





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**Step 7: Using partial fraction expansion, expand H(s) into**

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****



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**H.W: Challenge 1: Design a digital Butterworth filter satisfying the constraints**

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**With *T=1 sec using Impulse invariant method.***

**Ans:** 

**Challenge 2: Design a digital Butterworth filter satisfying the constraints**

****

**With *T=1 sec using Bilinear Transformation.***

***Ans: ***

***Challenge 3: Determine the system function H(z) of the lowest order Butterworth digital filter with the following specification.***

1. ***3db ripple in pass band ***
2. ***25db attenuation in stop band ***

***Ans: ***

***Challenge 4: Enumerate the various steps involved in the design of low pass digital Butterworth IIR filter. (ii) The specification of the desired low pass filter is***

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***Design a Butterworth digital filter using impulse invariant transformation.***

***Ans:***



***Design a chebyshev filter for the following specification using bilinear transformation.***

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***Solution:***

*Given data:*

*Pass band attenuation; Pass band frequency;*

*Stop band attenuation; Stops band frequency;*

**Step 1: Specifying the pass band and stop band attenuation in dB.**



**Step2. Choose *T* and determine the analog frequencies (i.e) Prewarp band edge frequency  
**

**Step3. To find order of the filter**





*Rounding the next higher integer value N=2*

**Step4. The poles of chebyshev filter can be determined by**



Where,

 And calculate a, b,, 













**Step.5 Find the denominator polynomial of the transfer function using above poles.**



**Step 6 : The numerator of the transfer function depends on the value of N.**

* If N is Even substitute s=0 in the denominator polynomial and divide the result by Find the value. This value is equal to numerator



**Step 7: The Transfer function is**



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**Step 8: Apply bilinear transformation with to obtain the digital filter**



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***Design a chebyshev filter for the following specification using impulse invariance method.***

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***Solution:***

*Given data:*

*Pass band attenuation; Pass band frequency;*

*Stop band attenuation; Stops band frequency;*

**Step 1: Specifying the pass band and stop band attenuation in dB.**



**Step2. Choose *T* and determine the analog frequencies (i.e) Prewarp band edge frequency  
**

**Step3. To find order of the filter**





*Rounding the next higher integer value N=2*

**Step4. The poles of chebyshev filter can be determined by**



Where,

 And calculate a, b,, 













**Step.5 Find the denominator polynomial of the transfer function using above poles.**



**Step 6 : The numerator of the transfer function depends on the value of N.**

* If N is Even substitute s=0 in the denominator polynomial and divide the result by Find the value. This value is equal to numerator



**Step 7: The Transfer function is**



****

**Step 8: Using partial fraction expansion, expand H(s) into**



**Step 9: Now transform analog poles {Pk} into digital poles {epkT} to obtain the digital filter**



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***H.W: Challenge 1: Design a chebyshev filter to meet the constraints by using bilinear transformation and assume sampling period T=1 sec.***

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***Solution:***

**Ans: **

*===============================================================* ***H.W: Convert the following analog filter with transfer function using bilinear transformation. Ans:*** **

***Find the order and poles of a low pass Butterworth filter that has 3dB bandwidth of 500Hz and attenuation of 40dB at 1kHz.***

*===============================================================*

**Filter design using frequency translation (HPF, BPF, BRF):**

A digital filter can be converted into a digital high pass, band stop or another digital filter. These transformations are given below.

|  |  |
| --- | --- |
| **Low pass to Low pass** | **Low pass to high pass** |
|  |  |
| **Low pass to Band pass** | **Low pass to Band Stop** |
|  |  |

***============================================================================***

***Analog Domain:***

*The frequency transformation can be used to design on LPF with different pass band frequency HPF,BPF and BSF from a normalized Low pass filter ΩC=1 rad/sec.*

|  |  |
| --- | --- |
| **Low pass to Low pass** | **Low pass to high pass** |
|  |  |
| **Low pass to band pass** | **Low pass to band stop** |
|  |  |

*=====================================================================*

**H:W: 1. Design a digital chebyshev filter **

**by using bilinear transformation and assume period T=1 sec.**

**Ans :**

****

**2. Enumerate the various steps involved in the design of low pass digital Butterworth IIR filter.**

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**Design Butterworth digital filter using impulse invariant transformation.**

**Ans:**

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**3. Design a chebyshev low pass filter with the specifications  ripple in the pass band ripple in the stop band, using (a) Bilinear transformation (b) Impulse invariance.**

**(a) Bilinear transformation:**

**Ans:**

****

**(b) Impulse invariance:**

**Ans:**

****

**4. Use the backward difference for the derivative to convert the analog low pass filter with system function.**

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**Ans: **

**5. For the analog transfer function determine H(z) using impulse invariant technique. Assume T=1sec.**

** Ans:  [May/Jue-2016]**

**6. Determine H(z) using the impulse invariant technique for the analog transfer function.**

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**Ans: **

**7. Using bilinear transformation obtain H (z) if  and T=0.1s. Ans:**

**8. Convert the analog filter with system function  into a digital IIR filter using bilinear transformation. The digital filter should have a resonant frequency of [Nov/Dec-2015]**



9. **A digital filter with a 3 dB bandwidth of 0.25π is to be designed from the analog filter whose system response is  . Use bilinear transformation and obtain H(z). [Nov/Dec-15]**



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**Prove that **

**Solution:**

**Solution:**

** **

** **